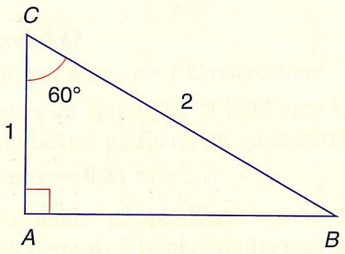
LØSNING AV TEST DEG SELV

KAPITTEL 1

Test I.A

a) Vi lager en trekant ABC med vinkler på 30°, 60° og 90°. Vi lar den korteste kateten ha lengden 1. Da blir hypotenusen 2:



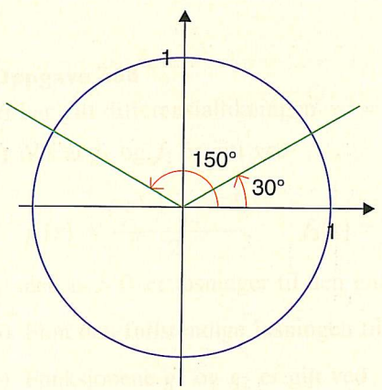
$$AB = \sqrt{2^2 - 1^2} = \sqrt{3}$$

$$\cos 60° = \frac{1}{2}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{1} = \sqrt{3}$$

b)



$$\sin 150° = \sin 30° = \frac{1}{2}$$

$$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\tan 150° = \frac{\sin 150°}{\cos 150°} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Test I.B

$$\sin 105° = \sin (60° + 45°)$$
$$= \sin 60° \cdot \cos 45° + \cos 60° \cdot \sin 45°$$
$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\cos 105° = \cos (60° + 45°)$$
$$= \cos 60° \cdot \cos 45° - \sin 60° \cdot \sin 45°$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\tan 105° = \frac{\sin 105°}{\cos 105°}$$
$$= \frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{2} - \sqrt{6}}{4}} = \frac{(\sqrt{2} + \sqrt{6})}{(\sqrt{2} - \sqrt{6})} \cdot \frac{(\sqrt{2} + \sqrt{6})}{(\sqrt{2} + \sqrt{6})}$$
$$= \frac{2 + 2 \cdot \sqrt{2}\sqrt{6} + 6}{2 - 6} = \frac{8 + 2\sqrt{4}\sqrt{3}}{-4}$$
$$= \frac{4(2 + \sqrt{3})}{-4} = -2 - \sqrt{3}$$

Test I.C

a) $\sin (x - 45°) + \sin (x + 45°)$
$- \cos (x + 30°) - \cos (x - 60°)$
$= \sin x \cos 45° - \cos x \sin 45°$
$+ \sin x \cos 45° + \cos x \sin 45°$
$- \cos x \cos 30° + \sin x \sin 30°$
$- \cos x \cos 60° - \sin x \sin 60°$

$$= 2 \sin x \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x - \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$$

$$= \left(\sqrt{2} + \frac{1}{2} - \frac{\sqrt{3}}{2}\right) \sin x + \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \cos x$$

$$= \frac{2\sqrt{2} + 1 - \sqrt{3}}{2} \sin x - \frac{\sqrt{3} + 1}{2} \cos x$$

b) $\sin^2 x + 1 + \cos^2 x$
$= (\sin^2 x + \cos^2 x) + 1 = 1 + 1 = 2$

c) $$\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{(\sin^2 x - \cos^2 x)} = 1$$

Test I.D

a) $3 \tan 2x = \sqrt{3}, \quad x \in [0, 360°\rangle$

$$\tan 2x = \frac{\sqrt{3}}{3}$$
$$2x = 30° + n \cdot 180°$$
$$x = 15° + n \cdot 90°$$

$L = \{15°, 105°, 195°, 285°\}$

b) $\sin (x - 30°) - \cos (x - 30°) = 0, \quad x \in [0, 360°\rangle$

$$\sin (x - 30°) = \cos (x - 30°)$$
$$\tan (x - 30°) = 1$$
$x - 30° = 45°$ eller $x - 30° = 225°$
$x = 75°$ eller $x = 255°$

c) $1 + \cos x = \sin^2 x, \qquad x \in [0, 360°\rangle$

$$1 + \cos x = 1 - \cos^2 x$$
$$\cos^2 x + \cos x = 0$$
$$\cos x (\cos x + 1) = 0$$
$\cos x = 0$ eller $\cos x + 1 = 0$
$x = \pm 90° + n \cdot 360°$ eller $x = 180° + n \cdot 360°$

Vi varierer n og velger ut de som passer med grunnmengden.

$L = \{90°, 180°, 270°\}$

Test I.E

$$\cos x = \frac{-12}{13}, \qquad x \in [90°, 180°\rangle$$

$$\sin x = \sqrt{1 - \cos^2 x} = \sqrt{\frac{169 - 144}{169}} = \frac{5}{13}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{5}{13}}{\frac{-12}{13}} = -\frac{5}{12}$$

$$\sin 2x = 2 \sin x \cos x = 2 \cdot \frac{5}{13} \cdot \frac{-12}{13} = -\frac{120}{169}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$
$$= \frac{144 - 25}{169} = \frac{119}{169}$$

$$\tan 2x = \frac{\frac{-120}{169}}{\frac{119}{169}} = -\frac{120}{119}$$

Vi bruker at $\cos x = \cos\left(2 \cdot \frac{x}{2}\right)$ og bruker regelen om cosinus til dobbelt vinkel.

$$\cos\left(2 \cdot \frac{x}{2}\right) = 1 - 2 \sin^2 \frac{x}{2}$$
$$2 \sin^2 \frac{x}{2} = 1 - \cos x$$
$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$
$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \frac{-12}{13}}{2}}$$
$$\sin \frac{x}{2} = \pm \sqrt{\frac{25}{26}}$$
$$\sin \frac{x}{2} = \pm \frac{5}{\sqrt{26}}$$

Test I.F

a) $f(x) = \sin^2 x + 2\cos^2 x$

$= (\sin^2 x + \cos^2 x) + \cos^2 x = 1 + \cos^2 x$

$f_{\text{maks}} = 1 + 1 = 2, \quad f_{\text{min}} = 1 + 0 = 1$

b) $f(x) = 2 + 2\sin 2x$

$f_{\text{maks}} = 2 + 2 \cdot 1 = 4, \quad f_{\text{min}} = 2 + 2 \cdot (-1) = 0$

Test I.G

a) $\cos x = \dfrac{AC}{CT} = \dfrac{AC}{4} \Rightarrow AC = 4\cos x$

b) $\sin x = \dfrac{AD}{AC}$

$AD = AC \cdot \sin x = 4\cos x \sin x = 2\sin 2x$

$AB = 8\cos x \sin x = 4\sin 2x$

$\cos x = \dfrac{CD}{AC} \Rightarrow CD = AC \cdot \cos x = 4\cos^2 x$

c) $A(x) = \dfrac{AB \cdot CD}{2} = \dfrac{8\sin x \cos x \cdot 4\cos^2 x}{2}$

$= 16\sin x \cos^3 x = 16\cos^3 x\sqrt{1 - \cos^2 x}$

$u = \cos x$

$A(u) = 16u^3 \cdot \sqrt{1 - u^2}$

Vi deriverer med produktregelen og får

$$A'(u) = 16 \cdot \left(3u^2\sqrt{1-u^2} + u^3 \cdot \frac{1}{2\sqrt{1-u^2}} \cdot (-2u)\right)$$

$$= 16 \cdot \left(\frac{3u^2\left(\sqrt{1-u^2}\right)^2 + u^3(-u)}{\sqrt{1-u^2}}\right)$$

$$= 16 \cdot \left(\frac{3u^2 \cdot (1-u^2) - u^4}{\sqrt{1-u^2}}\right)$$

$$= 16 \cdot \left(\frac{3u^2 - 3u^4 - u^4}{\sqrt{1-u^2}}\right)$$

$$= 16 \cdot \left(\frac{3u^2 - 4u^4}{\sqrt{1-u^2}}\right)$$

Vi finner høyeste verdi ved å sette $A'(u) = 0$:

$$A'(u) = 0$$

$$16 \cdot \left(\frac{3u^2 - 4u^4}{\sqrt{1-u^2}}\right) = 0$$

$$3u^2 - 4u^4 = 0$$

$$u^2(3 - 4u^2) = 0$$

$u^2 = 0$ eller $3 - 4u^2 = 0$

$u = 0$ eller $4u^2 = 3$

$u = 0$ eller $u^2 = \dfrac{3}{4}$

$u = 0$ eller $u = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$

Siden $2x \in \langle 0, 180\rangle$, har vi $x \in \langle 0, 90\rangle$.
Derfor må $\cos x = u \in \langle 0, 1\rangle$. Altså har vi

$u = \dfrac{\sqrt{3}}{2}$

$\cos x = \dfrac{\sqrt{3}}{2}$

$x = \pm 30° + n \cdot 360°$

$x = 30°$

Til slutt setter vi inn i $A(x)$ og finner arealet:

$A_{\text{maks}} = A(30°) = 16\sin 30° (\cos 30°)^3$

$= 16 \cdot \dfrac{1}{2} \cdot \left(\dfrac{\sqrt{3}}{2}\right)^3 = 16 \cdot \dfrac{(\sqrt{3})^3}{2 \cdot 2^3} = 3\sqrt{3}$